CSE | Computer Graphics Fall 2016 Midterm | UTA

NAME:

Prob #	1	2	3	4	5	6
Points	12	16	24	18	12	18

Time: 80 Minutes

NOTES:

a. **Credit is only given to the correct numerical values.**
b. **All numerical values must be calculated with three digits of accuracy after the decimal point.**
c. **Do not write on the back side of the papers.**

1. Points **A(-1,12)** and **B(60,6)** are given in a two dimensional world coordinate system. Find the coordinates of the points A and B on the screen after they have been mapped from window to viewport.

$x_{wmin} = -40$ $y_{wmin} = 5$ $x_{wmax} = 360$ $y_{wmax} = 15$

Normalized device coordinate of the viewport:

$x_{vmin} = 0.25$ $y_{vmin} = 0.1$ $x_{vmax} = 0.5$ $y_{vmax} = 0.6$

The origin of the screen coordinate system is defined in the **upper left** corner of the screen and the screen resolution is 800 by 600.
Use **truncation** to convert from float to integer.

Screen coordinates of point A after mapping are:

$\boldsymbol{S_x = \frac{0.5-0.25}{360-(-40)} = 0.000625,\ S_y = \frac{0.6-0.1}{15-5} = 0.05}$ **(4 points)**

$\boldsymbol{A_x = [0.25 + 0.000625(-1-(-40))] \times 800 = 219.5 \sim 219}$ **(2 points)**

$\boldsymbol{A_y = [0.1 + 0.05(15-12)] \times 600 = 150}$ **(2 points)**

Screen coordinates of point B after mapping are:

$\boldsymbol{B_x = [0.25 + 0.000625(60-(-40))] \times 800 = 250}$ **(2 points)**

$\boldsymbol{B_y = [0.1 + 0.05(15-6)] \times 600 = 330}$ **(2 points)**

2. Equations of plane P and line L are given as:

Plane P: $\mathbf{5x + 6y + 7z - 11 = 0}$

Line L : $\begin{cases} x(t) = 2 \\ y(t) = -3t \\ z(t) = 4t + 3 \end{cases}$

a. Find the point of intersection of line L with plane P.

Intersection point of line L and plane p is: _[2,6,-5]_ **(4 points)**

$5 \times 2 + 6 \times (-3t) + 7 \times (4t + 3) - 11 = 0 \Rightarrow t = -2$ **(4 points)**

⇨ $intersection: (2,6,-5)$

b. Find the equation of plane P after it has been translated by

$\boldsymbol{dx = 6\,, \quad dy = 9\,, \quad dz = -2}$

$\boldsymbol{new\ point: (8,15,-7)}$

⇨ $\mathbf{5x + 6y + 7z + C = 0}$

⇨ $\boldsymbol{C = -81}$ **(4 points)**

Equation of plane P after translation is:

___$5x + 6y + 7z - 81 = 0$___ **(4 points)**

3. The viewing parameters for a perspective projection are given as

VRP(WC)=(2,6,3) VPN(WC)=(6,0,0)
VUP(WC)=(0,3,4) PRP (VRC)=(3,6,-10)
u_{min} (VRC) = 1 u_{max} (VRC) = 11
v_{min} (VRC) = -1 v_{max} (VRC) = 3
n_{min} (VRC) = 10 n_{max} (VRC) = 40

a. Find the sequence of transformations which will transform this viewing volume into a standard perspective view volume which is bounded by the planes: x=z ; x=-z ; y=z ; y=-z ; z=1 (Complete the blank matrices)
b. Find the zmin after all transformations are done.

Matrix #2: Rx

1	0	0	0
0	1	0	0
0	0	1	0
0	0	0	1

Matrix #1: Translate

1	0	0	-2
0	1	0	-6
0	0	1	-3
0	0	0	1

Matrix #4: Rz (4 points)

0.6	0.8	0.0	0.0
-0.8	0.6	0.0	0.0
0.0	0.0	1.0	0.0
0.0	0.0	0.0	1.0

Matrix #3: Ry (4 points)

0.0	0.0	-1.0	0.0
0.0	1.0	0.0	0.0
1.0	0.0	0.0	0.0
0.0	0.0	0.0	1.0

Matrix #6 Shear (4 points)

1.0	0.0	-0.3	0.0
0.0	1.0	0.5	0.0
0.0	0.0	1.0	0.0
0.0	0.0	0.0	1.0

Matrix #5: Translate

1	0	0	-3
0	1	0	-6
0	0	1	10
0	0	0	1

Matrix #8 Scale (4 points)

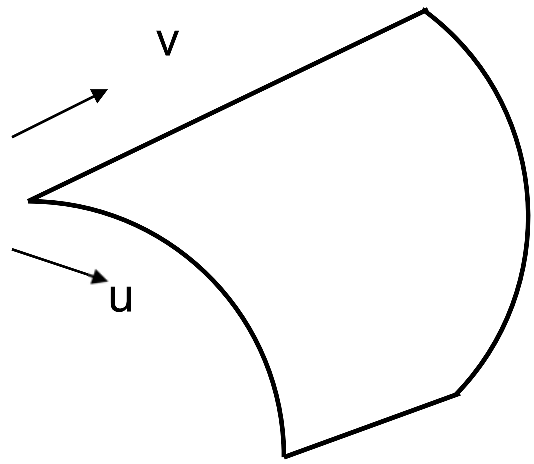
2	0.0	0.0	0.0
0.0	5	0.0	0.0
0.0	0.0	1.0	0.0
0.0	0.0	0.0	1.0

Matrix #7 Scale (4 points)

0.02	0.0	0.0	0.0
0.0	0.02	0.0	0.0
0.0	0.0	0.02	0.0
0.0	0.0	0.0	1.0

Zmin= 0.4 (4 points)

4. A curved surface is quadric in the u direction and linear in the v direction



The parametric equation of the curve corresponding to v=0 is given as:

$C(u) = 9\,u^2 + 4u - 2$

The parametric equation of the curve corresponding to v=1 is given as:

$C(u) = -7\,u^2 + 8u + 12$

Find the coefficient matrix C for this surface.

$$[u^2 \quad u \quad 1]\begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \\ c_{31} & c_{32} \end{bmatrix}\begin{bmatrix} v \\ 1 \end{bmatrix} = c_{11}u^2v + c_{12}u^2 + c_{21}uv + c_{22}u + c_{31}v + c_{32}$$

$$\overset{v=0}{\Longrightarrow} c_{12}u^2 + c_{22}u + c_{32} = 9u^2 + 4u - 2$$

$\Rightarrow c_{12} = 9, c_{22} = 4, c_{32} = -2$ **(6 points)**

$$\overset{v=1}{\Longrightarrow} c_{11}u^2 + 9u^2 + c_{21}u + 4u + c_{31} - 2 = -7u^2 + 8u + 12$$

$$\Rightarrow c_{11} = -16, c_{21} = 4, c_{31} = 14$$

$C = \begin{bmatrix} -16 & 9 \\ 4 & 4 \\ 14 & -2 \end{bmatrix}$ **(12 points)**

5. Equation of a cubic curve is given as:

C(t) = 6t³ +9t² -12 t +8

Find the numerical values of the Bezier geometry vector for this curve.

$C(t) = 6t^3 + 9t^2 - 12t + 8$

- ⇨ $C(0) = 8$
- ⇨ $C(1) = 11$

$C'(t) = 18t^2 + 18t - 12$

- ⇨ $C'(0) = -12$
- ⇨ $C'(1) = 24$

$G_H(x) = [8 \quad 11 \quad -12 \quad 24]$ **(6 points)**

$G_B(x) = \left[8 \quad 8 + \left(\frac{-12}{3}\right) \quad 11 - \left(\frac{24}{3}\right) \quad 11\right] = [8 \quad 4 \quad 3 \quad 11]$ **(6 points)**

6. Given the equation of a parametric surface

x(u,v) = $40u^2v + 20uv + 9$

y(u,v) = $8u^2 - 10uv + 11$

z(u,v) = $20u^2v - 10u^2 + 6$

Find the normal to this surface at u=0.5 and v=0.2

Hint: Find the tangect vectors in u and v directions and then find the cross product of these two vectors.

$$\begin{cases} \frac{\partial x}{\partial u} = 80uv + 20v \\ \frac{\partial y}{\partial u} = 16u - 10v \\ \frac{\partial z}{\partial u} = 40uv - 20u \end{cases}, \quad \begin{cases} \frac{\partial x}{\partial v} = 40u^2 + 20u \\ \frac{\partial y}{\partial v} = -10u \\ \frac{\partial z}{\partial v} = 20u^2 \end{cases}$$

$$\xRightarrow{u=0.5, v=0.2} \begin{cases} \frac{\partial x}{\partial u} = 12 \\ \frac{\partial y}{\partial u} = 6 \\ \frac{\partial z}{\partial u} = -6 \end{cases}$$

$$\xRightarrow{u=0.5, v=0.2} \begin{cases} \frac{\partial x}{\partial v} = 20 \\ \frac{\partial y}{\partial v} = -5 \\ \frac{\partial z}{\partial v} = 5 \end{cases}$$

- ⇨ $N = [0 \quad 180 \quad 180]\ or\ [0 \quad -180 \quad -180]$
- ⇨ $N = [0 \quad 0.7071 \quad 0.7071]\ or\ [0 \quad -0.7071 \quad -0.7071]$

Tangent vector in the u direction at point u=0.5 and v=0.2 is: [12 6 -6] **(6 points)**

Tangent vector in the v direction at point u=0.5 and v=0.2 is: [20 -5 5] **(6 points)**

Normal vector at u=0.5 and v=0.2: [0 0.707 0.707] or [0 -0.707 -0.707] **(6 points)**

$$R_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$M_{Hermite} = \begin{bmatrix} 2 & -3 & 0 & 1 \\ -2 & 3 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ 1 & -1 & 0 & 0 \end{bmatrix} \quad M_{Bezier} = \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

Rotate a vector around x axis until it lies in the xz plane

$$V = \begin{bmatrix} a \\ b \\ c \\ 1 \end{bmatrix} \qquad R_x = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{c}{\sqrt{b^2+c^2}} & \frac{-b}{\sqrt{b^2+c^2}} & 0 \\ 0 & \frac{b}{\sqrt{b^2+c^2}} & \frac{c}{\sqrt{b^2+c^2}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rotate a vector around y axis until it lies in the yz plane

$$V = \begin{bmatrix} a \\ b \\ c \\ 1 \end{bmatrix} \qquad R_y = \begin{bmatrix} \frac{c}{\sqrt{a^2+c^2}} & 0 & \frac{-a}{\sqrt{a^2+c^2}} & 0 \\ 0 & 1 & 0 & 0 \\ \frac{a}{\sqrt{a^2+c^2}} & 0 & \frac{c}{\sqrt{a^2+c^2}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rotate a vector around z axis until it lies in the yz plane

$$V = \begin{bmatrix} a \\ b \\ c \\ 1 \end{bmatrix} \qquad R_z = \begin{bmatrix} \frac{b}{\sqrt{a^2+b^2}} & \frac{-a}{\sqrt{a^2+b^2}} & 0 & 0 \\ \frac{a}{\sqrt{a^2+b^2}} & \frac{b}{\sqrt{a^2+b^2}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

How to convert a general perspective view volume into canonical perspective volume

Step 1: Translate VRP to origin
Step 2: Rotate VPN around x until it lies in the xz plane with positive z
Step 3: Rotate VPN around y until it aligns with the positive z axis.
Step 4: Rotate VUP around z until it lies in the yz plane with positive y
Step 5: Translate PRP (COP) to the origin
Step 6: Shear such that the center line of the view volume becomes the z axis
Step 7: Scale such that the sides of the view volume become 45 degrees
Step 8: Scale such that the view volume becomes the canonical perspective volume